# Quantum Harmonic Oscillator Ladder Operators

David G. Simpson

January 1, 2007

Here $a$ is the demotion (annililation, lowering) operator; and $a^\dagger$ is the promotion (creation, raising) operator for the quantum-mechanical simple harmonic oscillator.

## One Operator

$$\begin{aligned} a\,|n\rangle &= \sqrt{n}\,|n-1\rangle \\ a^\dagger\,|n\rangle &= \sqrt{n+1}\,|n+1\rangle \end{aligned}$$

## Two Operators

$$\begin{aligned} aa\,|n\rangle &= \sqrt{n(n-1)}\,|n-2\rangle \\ aa^\dagger\,|n\rangle &= (n+1)\,|n\rangle \\ a^\dagger a\,|n\rangle &= n\,|n\rangle \\ a^\dagger a^\dagger\,|n\rangle &= \sqrt{(n+1)(n+2)}\,|n+2\rangle \end{aligned}$$

## Three Operators

$$\begin{aligned} aaa\,|n\rangle &= \sqrt{n(n-1)(n-2)}\,|n-3\rangle \\ aaa^\dagger\,|n\rangle &= (n+1)\sqrt{n}\,|n-1\rangle \\ aa^\dagger a\,|n\rangle &= n^{3/2}\,|n-1\rangle \\ aa^\dagger a^\dagger\,|n\rangle &= (n+2)\sqrt{n+1}\,|n+1\rangle \\ a^\dagger aa\,|n\rangle &= (n-1)\sqrt{n}\,|n-1\rangle \\ a^\dagger aa^\dagger\,|n\rangle &= (n+1)^{3/2}\,|n+1\rangle \\ a^\dagger a^\dagger a\,|n\rangle &= n\sqrt{n+1}\,|n+1\rangle \\ a^\dagger a^\dagger a^\dagger\,|n\rangle &= \sqrt{(n+1)(n+2)(n+3)}\,|n+3\rangle \end{aligned}$$

## Four Operators

$$
\begin{aligned}
aaaa\,|n\rangle &= \sqrt{n(n-1)(n-2)(n-3)}\,|n-4\rangle \\
aaaa^\dagger\,|n\rangle &= (n+1)\sqrt{n(n-1)}\,|n-2\rangle \\
aaa^\dagger a\,|n\rangle &= n^{3/2}\sqrt{n-1}\,|n-2\rangle \\
aaa^\dagger a^\dagger\,|n\rangle &= (n+1)(n+2)\,|n\rangle \\
aa^\dagger aa\,|n\rangle &= (n-1)^{3/2}\sqrt{n}\,|n-2\rangle \\
aa^\dagger aa^\dagger\,|n\rangle &= (n+1)^2\,|n\rangle \\
aa^\dagger a^\dagger a\,|n\rangle &= n(n+1)\,|n\rangle \\
aa^\dagger a^\dagger a^\dagger\,|n\rangle &= (n+3)\sqrt{(n+1)(n+2)}\,|n+2\rangle \\
a^\dagger aaa\,|n\rangle &= (n-2)\sqrt{n(n-1)}\,|n-2\rangle \\
a^\dagger aaa^\dagger\,|n\rangle &= n(n+1)\,|n\rangle \\
a^\dagger aa^\dagger a\,|n\rangle &= n^2\,|n\rangle \\
a^\dagger aa^\dagger a^\dagger\,|n\rangle &= (n+2)^{3/2}\sqrt{n+1}\,|n+2\rangle \\
a^\dagger a^\dagger aa\,|n\rangle &= n(n-1)\,|n\rangle \\
a^\dagger a^\dagger aa^\dagger\,|n\rangle &= (n+1)^{3/2}\sqrt{n+2}\,|n+2\rangle \\
a^\dagger a^\dagger a^\dagger a\,|n\rangle &= n\sqrt{(n+1)(n+2)}\,|n+2\rangle \\
a^\dagger a^\dagger a^\dagger a^\dagger\,|n\rangle &= \sqrt{(n+1)(n+2)(n+3)(n+4)}\,|n+4\rangle
\end{aligned}
$$

## Five Operators

$$
\begin{aligned}
aaaaa\,|n\rangle &= \sqrt{n(n-1)(n-2)(n-3)(n-4)}\,|n-5\rangle \\
aaaaa^\dagger\,|n\rangle &= (n+1)\sqrt{n(n-1)(n-2)}\,|n-3\rangle \\
aaaa^\dagger a\,|n\rangle &= n^{3/2}\sqrt{(n-1)(n-2)}\,|n-3\rangle \\
aaaa^\dagger a^\dagger\,|n\rangle &= (n+1)(n+2)\sqrt{n}\,|n-1\rangle \\
aaa^\dagger aa\,|n\rangle &= (n-1)^{3/2}\sqrt{n(n-2)}\,|n-3\rangle \\
aaa^\dagger aa^\dagger\,|n\rangle &= (n+1)^2\sqrt{n}\,|n-1\rangle \\
aaa^\dagger a^\dagger a\,|n\rangle &= (n+1)n^{3/2}\,|n-1\rangle \\
aaa^\dagger a^\dagger a^\dagger\,|n\rangle &= (n+2)(n+3)\sqrt{n+1}\,|n+1\rangle \\
aa^\dagger aaa\,|n\rangle &= (n-2)^{3/2}\sqrt{n(n-1)}\,|n-3\rangle \\
aa^\dagger aaa^\dagger\,|n\rangle &= (n+1)n^{3/2}\,|n-1\rangle \\
aa^\dagger aa^\dagger a\,|n\rangle &= n^{5/2}\,|n\rangle \\
aa^\dagger aa^\dagger a^\dagger\,|n\rangle &= (n+2)^2\sqrt{n+1}\,|n+1\rangle \\
aa^\dagger a^\dagger aa\,|n\rangle &= (n-1)n^{3/2}\,|n-1\rangle \\
aa^\dagger a^\dagger aa^\dagger\,|n\rangle &= (n+2)(n+1)^{3/2}\,|n+1\rangle \\
aa^\dagger a^\dagger a^\dagger a\,|n\rangle &= n(n+2)\sqrt{(n+1)}\,|n+1\rangle \\
aa^\dagger a^\dagger a^\dagger a^\dagger\,|n\rangle &= (n+4)\sqrt{(n+1)(n+2)(n+3)}\,|n+3\rangle \\
a^\dagger aaaa\,|n\rangle &= (n-3)\sqrt{n(n-1)(n-2)}\,|n-3\rangle \\
a^\dagger aaaa^\dagger\,|n\rangle &= (n-1)(n+1)\sqrt{n}\,|n-1\rangle \\
a^\dagger aaa^\dagger a\,|n\rangle &= (n-1)n^{3/2}\,|n-1\rangle \\
a^\dagger aaa^\dagger a^\dagger\,|n\rangle &= (n+2)(n+1)^{3/2}\,|n+1\rangle \\
a^\dagger aa^\dagger aa\,|n\rangle &= (n-1)^2\sqrt{n}\,|n-1\rangle \\
a^\dagger aa^\dagger aa^\dagger\,|n\rangle &= (n+1)^{5/2}\,|n+1\rangle \\
a^\dagger aa^\dagger a^\dagger a\,|n\rangle &= n(n+1)^{3/2}\,|n+1\rangle \\
a^\dagger aa^\dagger a^\dagger a^\dagger\,|n\rangle &= (n+3)^{3/2}\sqrt{(n+1)(n+2)}\,|n+3\rangle \\
a^\dagger a^\dagger aaa\,|n\rangle &= (n-1)(n-2)\sqrt{n}\,|n-1\rangle \\
a^\dagger a^\dagger aaa^\dagger\,|n\rangle &= n(n+1)^{3/2}\,|n+1\rangle \\
a^\dagger a^\dagger aa^\dagger a\,|n\rangle &= n^2\sqrt{n+1}\,|n+1\rangle \\
a^\dagger a^\dagger aa^\dagger a^\dagger\,|n\rangle &= (n+2)^{3/2}\sqrt{(n+1)(n+3)}\,|n+3\rangle \\
a^\dagger a^\dagger a^\dagger aa\,|n\rangle &= n(n-1)\sqrt{n+1}\,|n+1\rangle \\
a^\dagger a^\dagger a^\dagger aa^\dagger\,|n\rangle &= (n+1)^{3/2}\sqrt{(n+2)(n+3)}\,|n+3\rangle \\
a^\dagger a^\dagger a^\dagger a^\dagger a\,|n\rangle &= n\sqrt{(n+1)(n+2)(n+3)}\,|n+3\rangle \\
a^\dagger a^\dagger a^\dagger a^\dagger a^\dagger\,|n\rangle &= \sqrt{(n+1)(n+2)(n+3)(n+4)(n+5)}\,|n+5\rangle
\end{aligned}
$$